$\qquad$ Date $\qquad$

## Advanced Algebra

Sequence and Series
Unit 1: Review
Write the recursive sequence for the given tables
Arithmetic

| $n$ | $U_{n}$ |
| :--- | :--- |
| 2 | 1 |
| 4 | 5 |
| 7 | 11 |

Shifted Geometric

| $n$ | $U_{n}$ |
| :--- | :--- |
| 0 | 8 |
| 1 | 19 |
| 2 | 41 |
| 3 | 85 |

I can recognize a geometric, arithmetic and shifted recursive sequence.
State what type of sequence the following are:
4) $y=5 x+6$
5) $y=\frac{1}{2} *(3)^{x}$

I can write a sequence given a graph


I can write a geometric recursive sequence. I can write a direct formula a geometric.
Example: Sally bought a car for $\$ 35,000$. The car depreciates $6 \%$ per year. What will the car be worth in 10 years.
$\mathrm{U}_{0}=35,000$
$U_{n}=(1-.06) * U_{(n-1)} \quad$ We can make this a direct formula by $y=U_{0} * r^{x}$ So this would be $y=35,000(.94)^{10}$
This gives us $\$ 18,851.53$. The Key to all depreciation problems is (1-r). All depreciation problems are yearly.

You try....
7) John bought a boat. The cost of the boat is $\$ 45,000$. The boat depreciates $5 \%$ per year. Write the recursive and the direct formula for this. What will the boat be worth in 10 years? When will the boat be worth around $\$ 22,000$. (You should be using your table for this question)
8) A new car was purchased for $\$ 13,000$. The car will depreciate by $8 \%$ per year. After how many years will the car be worth $\$ 8,000$ ?

## Another example of simple geometric is appreciation.

Beth deposited $\$ 800$ into an account that earns $8 \%$ APR. She leaves it in the account. What will the account be worth in 12 years?

The key to appreciation problems is (1+r)....remember sometimes we need to change the $r$ by dividing it when we see that we are changing the compounding period.

Solution:
$\mathrm{U}_{0}=800$
$U_{n}=(1+.08) * U_{(n-1)}$ As a direct formula we can write $y=800(1+.08)^{12}$ This gives us a value of $\$ 2014.54$
We know if we read something that says quarterly, monthly, weekly, we need to divide the APR by that amount and adjust the exponent as well. However it is still a simple geometric equation.

You try the following: Please write the direct formula for these problems.
9) Beth took out a loan for $\$ 13,000$. The APR on the loan was $8 \%$. What will the balance be in 8 years? You should be writing a simple direct formula for this problem where 8 is the exponent.
10) Beth took out a loan for $\$ 13,000$. The APR on the loan was $8 \%$ compounded quarterly. What will the balance be in 8 years? Remember in this situation you need to divide the rate by 4 and the exponent will now be 32 ...
11) Beth took out a loan for $\$ 13,000$. The APR on the loan was $8 \%$ compounded monthly. What will the balance be in 8 year? Remember in this situation you need to divide the rate by 12 and the exponent will now be 96

I can work with a shifted geometric sequence. The key point is we can write the sequence and then we want to use our calculator.

Beth took out a loan for $\$ 13,000$. The APR on the loan was $8 \%$ compounded quarterly. She also makes payments of $\$ 500$ every quarter as well. Write the recursive and how long will it take for her to pay it off?
$\mathrm{U}_{0}=13,000$
$U_{n}=\left(1+\frac{.08}{4}\right) * U(n-1)-500$ I need to use my calculator for this. I am going to put it in sequence mode. Remember that $u$ is $2^{\text {nd }} 7$. Then just go to table. I see that it will take 38 total month. 37 months paying 500 and 1 final month of paying 31.78. I got this from the table.

You try. I can write a shifted geometric sequence.
12) You deposit money into an account. The initial deposit was $\$ 10,000$. The APR on the account is $7 \%$ APR compounded quarterly. You are going to deposit additional money into this account. You will deposit $\$ 100$ every quarter as well. ( Look at the other example. Instead of a minus you will have $a+100$ ). What will the account be worth in 12 year. (You are looking for 48 in your table.)

## I can find Limit:

13) Find the limit of the following sequences:
$U_{0}=1000$
$U_{n=} .825^{*} \mathrm{U}_{(\mathrm{n}-1)}+32$
$n \geq 1$
14) A person takes out a loan for $\$ 500$. The APR on the loan is $14.4 \%$ compounded monthly. The person also makes $\$ 50$ payments every month. Write the shifted geometric sequence for this scenario and find out how many months it take to pay it off.
15) Given $\sum_{n=1}^{55} 6 n-5$

What are the first five terms of the sequence?

| Term 1 | Term 2 | Term 3 | Term 4 | Term 5 |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |

What is the sum of the first 55 terms?
16) Given $-1+1+3+5+\ldots .17$ What is the sum of this closed sequence?
17) Given $-3+2+7+12+\ldots 37$ What is the sum of this closed sequence?
18) What is the sum of the first 6 terms below?

Given: $\left\{\begin{array}{c}U_{0}=8 \\ U_{n}=2 * U_{(n-1)} \\ n \geq 1\end{array}\right.$
What is the $\mathrm{S}_{6}$ ?

## High Challenge Problems:

19) Joseph took out a loan for $\$ 35,000$. The APR on the loan was $7.4 \%$ compounded monthly. He makes $\$ 450$ payments every month.
a) Write the recursive sequence for this scenario.
b) How long will it take him to pay off the loan?
c) What is the exact amount of his last months payment?
d) How much in total did he pay for the car?
20) The Sum of the first 10 terms of an arithmetic series is 245 . The sum of the first 20 terms of the same arithmetic series is 790 .

Calculate the first term and the common difference
21) The sum of the first ten terms of an arithmetic sequence is 95 , and the sum of the first 20 terms of the same arithmetic sequence is 290 .
Calculate the first term and the common difference.
22) Given the geometric sequence $y=.5(20)^{x}$, Find the sum of the first 10 terms. What is the limit of the series?
23) The sum of the first thirty terms of an arithmetic sequence is 3,660 and the sum of the first 70 terms of the same arithmetic series is 19,740.
Calculate the first term and the common difference.

