Name_______________

Date_______________

Advanced Algebra

Partial Sums of a Geometric Series

Assignment #11

A partial sum of a geometric series is given by the explicit formula $S_n = \frac{U_1(1-r^n)}{(1-r)}$; where n is the number of terms. U_1 is the first term and r is the common ratio. Note $r \neq 1$

Example:

Given $\begin{cases} U_1 = 40 \\ U_n = .6 * U_{(n-1)} \\ n \geq 2 \end{cases}$ Find the following:

$\frac{40(1-.6^5)}{(1-.6)} =$

a) S_5
b) S_{15}
c) S_{25}
d) What does the limit of the Series appear to be?

$\frac{40(1-.6^{15})}{(1-.6)} =$

$\frac{40(1-.6^{25})}{(1-.6)} =$

1) Consider the sequence $U_1 = 8$ and $U_n = .5*U_{(n-1)}$; where $n \geq 2$; Find
   a) $\sum_{n=1}^{10} U_n$
   b) $\sum_{n=1}^{20} U_n$
   c) $\sum_{n=1}^{30} U_n$
   d) What does ~~the limit~~ of the series appear to be?

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$S_{10} = \frac{8(1-.5^{10})}{(1-.5)} = 15.98$

$S_{20} = \frac{8(1-.5^{20})}{(1-.5)} = 15.99$

$S_{30} = \frac{8(1-.5^{30})}{(1-.5)}$

2) Find the sum of the following geometric series:
   12+6+3+1.5+...to 10 terms

$\frac{6(1-.5^{20})}{(1-.5)}$

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3) Find the sum of the following geometric series
   3+9+27+...to 8 terms

$\frac{9(1-3^8)}{(1-3)}$

29,520

$S_{16} = \frac{4(1-2^{16})}{(1-2)}$ 524,280

4) Find the sum of the following geometric series
2+4+8+… to 16 terms

5) Let S_n be the sum of an infinite geometric sequence such that $S_1 = 3$ and $S_2 = 4$
   a) State the first term U_1
   b) Calculate the common ratio
   c) Calculate U_5

$U_1 = 3$

$3 + 3r = 4$

$3r = 1$

b) $r = \frac{1}{3}$

c) $y = 9\left(\frac{1}{3}\right)^5$

.037037

6) Suppose you begin a job with an annual salary of \$17,500. Each year, you can expect a 4.2% raise.
   a) What is your salary in the 10th year after you start the job?
   b) What is the total amount you can earn in 10 years? Asking you to sum up the 10 terms 220,972.65
   c) How long must you work at this job before your total earnings exceed \$1million? 29 years

a) $y = 17{,}500(1 + .042)^{10}$ 26406.76

$U_1 = 18{,}235$

b) $\frac{18235(1-1.042^{10})}{(1-1.042)}$

7) Each year a salesperson is paid a bonus which is banked into the same account. It earns an APR of 6%. The recursive formula can be modeled by the following:

$$\begin{cases} U_0 = 2000 \\ U_n = (1.06) * U_{(n-1)} \\ n \geq 1 \end{cases}$$

Find the bank account balance after 10 years. That is find S_{10}

c) $1{,}000{,}000 = \frac{18235(1-1.042^x)}{(1-1.042)}$

$\frac{-42000}{18235} = \frac{18235(1-1.042^x)}{18235}$

$-2.30326 = 1 - 1.042^x$
$-1 \qquad -1$

$-3.30326 = -1.042^x$

$3.30326 = 1.042^x$

$\log_{1.042} 3.30326$

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