

Name:

Advanced Algebra

Unit 5: Polynomials: Assignment #11

Factoring higher level polynomials:

For the following problems factor out the Prompt: Greatest Common Factor

1) $14x^2 + 8x + 72$

$$2(7x^2 + 4x + 36)$$

2) $3x^4 - 12x^3$

$$3x^3(x-4)$$

3) $-35x^3 + 28x^2 + 7x$

$$7x(-5x^2 + 4x + 1)$$

4) $24x^4 - 6x$

$$6x(4x^3 - 1)$$

5) $39x^5 + 13x^3 - 78x^2$

$$13x^2(3x^3 + x - 6)$$

6) $145x^9 - 29$

$$29(5x^9 - 1)$$

7) $6x^6 - 3x^4 - 9x^2$

$$3x^2(2x^4 - x^2 - 3)$$

8) $72x^9 + 15x^6 + 9x^3$

$$3x^3(24x^6 + 5x^3 + 3)$$

I can recognize Special patterns :

Name of Pattern	Example
Sum and Difference: $(a+b)(a-b) = a^2 - b^2$	$(x+3)(x-3) = x^2 - 9$
Square of a Binomial: $(a+b)^2 = a^2 + 2ab + b^2$ $(a-b)^2 = a^2 - 2ab + b^2$	$(y+4)^2 = y^2 + 8y + 16$ $(3t^2-2)^2 = 9t^4 - 12t^2 + 4$
Cube of a binomial: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	$(x+1)^3 = x^3 + 3x^2 + 3x + 1$ $(p-2)^3 = p^3 - 6p^2 + 12p - 8$
Sum of two Cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	$x^3 + 8 = (x+2)(x^2 - 2x + 4)$
Difference of two Cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$	$8x^3 - 1 = (2x-1)(4x^2 + 2x + 1)$

* Key point IS

① Do both Terms have A cube Root?

② If 'yes' Find the cube root of each

Classwork: Prompt: Use the Sum or Difference of Cubes to factor the polynomial. The state the Root(s)

Sum of two Cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	$x^3 + 8 = (x+2)(x^2 - 2x + 4)$
Difference of two Cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$	$8x^3 - 1 = (2x-1)(4x^2 + 2x + 1)$

③ Follow "minus" pattern
"plus" pattern

<p>1) $x^3 - 8$ $\begin{matrix} \uparrow & \uparrow \\ x & 2 \end{matrix}$ <i>x is "like a"</i> <i>2 is "like b"</i> Follow Difference Pattern $(x-2)(x^2 + 2x + 4)$</p>	<p>2) $x^3 + 64$ $\begin{matrix} \uparrow & \uparrow \\ x & 4 \end{matrix}$ Follow the Sum Pattern $(x+4)(x^2 - 4x + 16)$</p>
Root(s) 2	Root(s) -4

<p>3) $216x^3 + 1$ $\begin{matrix} \uparrow & \uparrow \\ 6x & 1 \end{matrix}$ Follow sum pattern $(6x+1)(36x^2 - 6x + 1)$</p>	<p>4) $125x^3 - 8$ $\begin{matrix} \uparrow & \uparrow \\ 5x & 2 \end{matrix}$ Follow Minus Pattern $(5x-2)(25x^2 + 10x + 4)$</p>
Root(s) $-\frac{1}{6}$	Root(s) $\frac{2}{5}$

<p>5) $1000x^3 + 27$ $\begin{matrix} \uparrow & \uparrow \\ 10x & 3 \end{matrix}$ Follow Sum Pattern $(10x+3)(100x^2 - 30x + 9)$</p>	<p>6) $27x^3 + 216$ $\begin{matrix} \uparrow & \uparrow \\ 3x & 6 \end{matrix}$ Follow sum Pattern $(3x+6)(9x^2 - 18x + 36)$</p>
Root(s) $-\frac{3}{10}$	Root(s) -2

<p>7) $32x^3 - 4$ $\begin{matrix} \uparrow & \uparrow \\ 2x & 1 \end{matrix}$ $4(8x^3 - 1)$ Follow minus pattern $4(2x-1)(4x^2 + 2x + 1)$</p>	<p>8) $2x^3 + 54$ $2(x^3 + 27)$ $\begin{matrix} \uparrow & \uparrow \\ x & 3 \end{matrix}$ $2(x+3)(x^2 - 3x + 9)$</p>
Root(s) $\frac{1}{2}$	Root(s) -3

Prompt: Grouping: Factor the following by grouping. Then state the root(s)

1) $x^3 + x^2 + x + 1$

$$(x^3 + x^2) + (x + 1)$$

$$x^2(x + 1) + 1(x + 1)$$

$$(x + 1)(x^2 + 1)$$

Root(s) -1

2) $10x^3 + 20x^2 + x + 2$

$$(10x^3 + 20x^2) + (x + 2)$$

$$10x^2(x + 2) + 1(x + 2)$$

$$(10x^2 + 1)(x + 2)$$

Root(s) -2

3) $x^3 + 3x^2 + 10x + 30$

$$(x^3 + 3x^2) + (10x + 30)$$

$$x^2(x + 3) + 10(x + 3)$$

$$(x + 3)(x^2 + 10)$$

Root(s) -3

4) $x^3 - 2x^2 + 4x - 8$

$$(x^3 - 2x^2) + (4x - 8)$$

$$x^2(x - 2) + 4(x - 2)$$

$$(x - 2)(x^2 + 4)$$

Root(s) 2

5) $2x^3 - 5x^2 + 18x - 45$

$$(2x^3 - 5x^2) + (18x - 45)$$

$$x^2(2x - 5) + 9(2x - 5)$$

$$(2x - 5)(x^2 + 9)$$

Root(s) 2.5

6) $-2x^3 - 4x^2 - 3x - 6$

$$(-2x^3 - 4x^2) + (-3x - 6)$$

$$-2x^2(x + 2) - 3(x + 2)$$

$$(x + 2)(-2x^2 - 3)$$

Root(s) -2

7) $3x^3 - 6x^2 + x - 2$

$$(3x^3 - 6x^2) + (x - 2)$$

$$3x^2(x - 2) + 1(x - 2)$$

$$(x - 2)(3x^2 + 1)$$

Root(s) 2

8) $2x^3 - x^2 + 2x - 1$

$$(2x^3 - x^2) + (2x - 1)$$

$$x^2(2x - 1) + 1(2x - 1)$$

$$(2x - 1)(x^2 + 1)$$

Root(s) $\frac{1}{2}$

9) $3x^3 - 2x^2 - 9x + 6$

$$(3x^3 - 2x^2) + (-9x + 6)$$

$$x^2(3x - 2) - 3(3x - 2)$$

$$(3x - 2)(x^2 - 3)$$

Root(s)

$\frac{2}{3}$ And $\pm\sqrt{3}$

Notes:

- ① Group first 2 terms
- ② Group second 2 terms
- ③ Look for GCF
- ④ Factor out GCF

Quadratic Form: Factor the polynomials: Prompt: Use special quadratic ideas to factor... State Root(s)

1) $16x^4 - 1$

2) $x^4 + 3x^2 + 2$

3) $x^4 - 81$

Factored $(4x^2 - 1)(4x^2 + 1)$

Root(s) $\pm \sqrt{\frac{1}{4}}$

Factored $(x^2 + 2)(x^2 + 1)$

NO ROOTS

Factored $(x^2 - 9)(x^2 + 9)$

Roots $\pm \sqrt{3}$

4) $81x^4 - 256$
 $(9x^2 - 16)(9x^2 + 16)$

5) $4x^4 - 5x^2 - 9$

6) $x^4 + 10x^2 + 16$

Factored $(3x - 4)(3x + 4)(9x^2 + 16)$

Roots $\frac{4}{3}$ And $-\frac{4}{3}$

Factored See paper below

Factored $(x^2 + 8)(x^2 + 2)$

7) $81 - 16x^4$

8) $32x^6 - 2x^2$
 $2(16x^6 - x^2)$

9) $6x^5 - 51x^3 - 27x$

Factored $(9 - 4x^2)(9 + 4x^2)$

Factored $2(4x^2 - 1)(4x^2 + 1)$

Roots $\pm \sqrt{\frac{1}{4}}$

Factored See paper below

Prompt: Mixed Practice: For the following problems, factor the problem completely.

Ideas: Look to factor out the GCF
 Is there a special pattern that you recognize?
 Once you make it quadratic, can you factor that more?
 Try to fit terms in the box

9) $3x^2 + 11x + 6$
 $(3x + 2)(x + 3)$

10) $x^3 - 4x^2 + 4x - 16$
 $(x^3 - 4x^2) + 4x - 16$
 $x^2(x - 4) + 4(x - 4)$
 $(x - 4)(x^2 + 4)$

11) $125x^3 - 216$
 $\nearrow \nearrow$
 $5x \quad 6$
 $(5x - 6)(25x^2 + 30x + 36)$

12) $2x^7 - 32x^3$
 $2x^3(x^4 - 16)$
 $2x^3(x^2 - 4)(x^2 + 4)$

13) $2x^5 + 4x^4 - 4x^3 - 8x^2$
 $2x^2(x^3 + 2x^2 - 2x - 4)$
 $[(x^3 + 2x^2) - 2(x + 2)]$
 $x^2(x + 2) - 2(x + 2)$
 $2x^2(x + 2)(x^2 - 2)$

14) $2x^3 - 32x$
 $2x(x^2 - 16)$
 $2x(x - 4)(x + 4)$
 Roots $0, 4$ And -4

Roots: $0, -2, \pm \sqrt{2}$

#5 $4x^4 - 5x^2 - 9$

~~| | | |
|----|-----|---|
| | -36 | |
| -9 | | 4 |
| | -5 | |~~

	$4x^2$	-9
x^2	$4x^4$	$-9x^2$
1	$4x^2$	-9

$$(4x^2 - 9)(x^2 + 1)$$

$$(2x-3)(2x+3)(x^2+1)$$

9

$6x^5 - 51x^3 - 27x$

~~| | | |
|-----|------|---|
| | -162 | |
| -54 | | 3 |
| | -51 | |~~

	$3x^2$	-27
$2x^2$	$6x^4$	$-54x^2$
1	$3x^2$	-27

$$(3x^2 - 27)(2x^2 + 1)$$

$$3(x^2 - 9)$$

$$3(x-3)(x+3)(2x^2+1)$$