Name_____

Date_____

Advanced Algebra

Unit 6: Finding the Inverse of a 2 by 2 Matrix

Unit 6 Assignment #12

You will use this assignment to complete the next assignment #13

Find the inverse of the following matricies. Matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ will be your inverse matrix.

We know that $[A]^{-1} * [A]$ is the equivalent to 1 in a 2 by 2 matrix. Just like we know $\frac{1}{6}*6=1$

You need to set up the system of equations to solve these matricies. This involves the skill of

A) **Multiplying matricies**
B) **Solving a 2 by 2 system with the skill of elimination.**

1) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 & -9 \\ -1 & 4 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$3a-b=1$ (4)
$-9a+4b=0$

$12a-4b=4$
$-9a+4b=0$
$3a=4$

$a=\frac{4}{3}$

$b=3$

$3c-d=0$ (4)
$-9c+4d=1$

$12c-4d=0$
$-9c+4d=1$
$3c=1$

$c=\frac{1}{3}$

$1-d=0$

$d=1$

$\begin{bmatrix} \frac{4}{3} & 3 \\ \frac{1}{3} & 1 \end{bmatrix}$

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2) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a+2b=1$ (2)
$-2a+b=0$

$2a+4b=2$
$-2a+b=0$

$b=\frac{2}{5}$

$a=\frac{1}{5}$

$c+2d=0$ (2)
$-2c+d=1$

$2c+4d=0$
$-2c+d=1$
$5d=1$

$d=\frac{1}{5}$

$c=-\frac{2}{5}$

$\begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$

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3) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & -4 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$3a+5b=1$ (2)
$2a-4b=0$ (3)

$6a+10b=0$
$-6a+12b=3$
$22b=3$

$b=\frac{3}{22}$

$3a=\frac{12}{22}$

$a=\frac{4}{22}$

$3c+5d=0$ (2)
$2c-4d=1$ (3)

$6c+10d=0$
$-6c+12d=3$
$22d=-3$

$d=-\frac{3}{22}$

$3c=\frac{15}{22}$

$c=\frac{5}{22}$

$\begin{bmatrix} \frac{4}{22} & \frac{3}{22} \\ \frac{5}{22} & -\frac{3}{22} \end{bmatrix}$

$3a = \frac{15}{33} \cdot \frac{1}{3}$ $\frac{5}{33}$

4) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 9 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$3a + 9b = 1$ (2)
$2a - 5b = 0$ (-3)

$6a + 18b = 2$
$-6a + 15b = 0$

$33b = 2$
$b = \frac{2}{33}$

$3c + 9d = 0$ (2)
$2c - 5d = 1$ (-3)

$6c + 18d = 0$
$-6c + 15d = -3$
$33d = -3$
$d = -\frac{3}{33}$

$3c = \frac{27}{33} \times \frac{1}{3}$
$c = \frac{9}{33}$

$\begin{bmatrix} \frac{5}{33} & \frac{2}{33} \\ \frac{9}{33} & -\frac{3}{33} \end{bmatrix}$

5) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 5 & -4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$5a + b = 1$ (4)
$-4a + 3b = 0$ (5)

$20a + 4b = 4$
$-20a + 15b = 0$

$19b = 4$
$b = \frac{4}{19}$

$5a = \frac{15}{19}$
$a = \frac{3}{19}$

$5c + 1d = 0$ (4)
$-4c + 3d = 1$ (5)

$20c + 4d = 0$
$-20c + 15d = 5$

$5c = -\frac{5}{19}$
$c = -\frac{1}{19}$
$d = \frac{5}{19}$

6) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 7 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$4a + 7b = 1$ (3)
$-3a + 12b = 0$ (4)

$12a + 21b = 3$
$-12a + 48b = 0$
$69b = 3$
$b = \frac{3}{69}$

$4a = \frac{48}{69}$
$a = \frac{12}{69}$

$4c = \frac{-28}{69}$
$c = -\frac{7}{69}$

$12c + 21d = 0$
$-12c + 48d = 4$
$69d = 4$
$d = \frac{4}{69}$

$4c + 7d = 0$ (3)
$-3c + 12d = 1$ (4)

$\begin{bmatrix} \frac{12}{69} & \frac{3}{69} \\ -\frac{7}{69} & \frac{4}{69} \end{bmatrix}$

7) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$3a - 2b = 1$
$4a + 3b = 0$

$12a - 8b = 4$
$-12a - 9b = 0$
$b = -\frac{4}{17}$
$a = \frac{3}{17}$

$3c - 2d = 0$ (4)
$4c + 3d = 1$ (3)

$12c - 8d = 0$
$-12c - 9d = 3$
$-17d = 3$
$d = -\frac{3}{17}$

$c = \frac{2}{17}$

8) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$5a + 4b = 1$ (2)
$-2a + 3b = 0$ (5)

$10a + 8b = 0$
$-10a + 15b = 0$
$23b = 2$
$b = \frac{2}{23}$

$5c + 4d = 0$ (2)
$-2c + 3d = 1$ (5)

$10c + 8d = 0$
$-10c + 15d = 5$
$23d = 5$
$d = \frac{5}{23}$

$\begin{bmatrix} \frac{3}{23} & \frac{2}{23} \\ -\frac{4}{23} & \frac{5}{23} \end{bmatrix}$

9) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 7 & 2 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$7a + 3b = 1$ (2)
$2a - 5b = 0$ (7)

$14a + 6b = 2$
$-14a + 35b = 0$
$41b = 2$
$b = \frac{2}{41}$

$7c + 3d = 0$ (5)
$2c - 5d = 1$ (3)

$35c + 15d = 0$
$6c - 15d = 3$
$41c = 3$
$c = \frac{3}{41}$

$\begin{bmatrix} \frac{5}{41} & \frac{2}{41} \\ \frac{3}{41} & -\frac{7}{41} \end{bmatrix}$