Assignment # T8

Adv. Algebra – Matrices – Solving Using Matrix Inverse (calculator assist)

Name:______________________________ Hr:___

ONE: Turn the following system into a matrix equation:

$$-x-5y-5z=2$$
$$4x-5y+4z=19$$
$$x+5y-z=-20$$

$$\begin{bmatrix} -1 & -5 & -5 \\ 4 & -5 & 4 \\ 1 & 5 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 19 \\ -20 \end{bmatrix}$$

TWO: Using inverse matrices solve for x,y,z in the systems of equations above.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [A]^{-1}\begin{bmatrix} 2 \\ 19 \\ -20 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -3 \\ 3 \end{bmatrix}$$

THREE: Use matrix inverse to solve for the unknown values in the following system of equations.

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 a +  b - 2c +  d + 3e -  f  =   4
2a -  b +  c + 2d +  e - 3f  =  20
 a + 3b - 3c -  d + 2e +  f  = -15
5a + 2b -  c -  d + 2e +  f  = - 3
-3a -  b + 2c + 3d +  e + 3f  =  16
4a + 3b +  c - 6d - 3e - 2f  = -27
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$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ 2 \\ -1 \end{bmatrix}$$

FOUR: the following represents some quadratic equation $f(x) = ax^2 + bx + c$

a. Using the information $f(3) = 15.7, f(5) = 49.3$ & $f(10) = 185.8$ set up a system of equations.

$$9a + 3b + c = 15.7$$
$$25a + 5b + c = 49.3$$
$$100a + 10b + c = 185.8$$

b. With matrix inverse solve for a,b,c.

$$\begin{bmatrix} 9 & 3 & 1 \\ 25 & 5 & 1 \\ 100 & 10 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15.7 \\ 49.3 \\ 185.8 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = [A^{-1}] \cdot \begin{bmatrix} 15.7 \\ 49.3 \\ 185.8 \end{bmatrix}$$

c. Thus, determine the equation of $f(x)$.

$$\boxed{1.5x^2 + 4.8x + -12.2}$$

FIVE: Jessica works at the bookstore at the Hamline University campus. She is very disorganized but has a great memory. Her boss sent her the email below on September 2nd, two days before the start of classes.

Jessica,

need to know how many of the following titles were sold yesterday so I know what books need to be restocked:

- Just War Theory (Walzer), \$14
- War and Peace (Tolstoy), \$18.50
- Grapes of Wrath (Steinbeck) \$23.25

Let me know,

Lynda

Let a = WAR Theory
b = WAR And Peace
c = Grapes

Jessica didn't write anything down or keep any receipts, but she remembers the following facts.

1. She sold 50 of those three titles in total.
2. She made a total of 909 dollars from those books.
3. She sold 22 more copies of *War and Peace* than *Grapes of Wrath.*

Task: Let J, W, and G represent the number of each title sold. Set up and use matrices to help Jessica answer Lynda's email.

$$a + b + c = 50$$

$$14a + 18.50b + 23.25c = 909$$

$$0a - b + c = -22$$

$c + 22 = b$

$$\begin{bmatrix} 1 & 1 & 1 \\ 14 & 18.50 & 23.25 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 50 \\ 909 \\ -22 \end{bmatrix}$$

$$[A^{-1}] \cdot \begin{bmatrix} 50 \\ 909 \\ -22 \end{bmatrix} \quad \begin{bmatrix} 12 \\ 30 \\ 8 \end{bmatrix}$$

War Theory = 12 copies
War Peace = 30 copies
Grapes = 8 copies

SIX: There are four boxes labeled A,B,C,D.

- The weight of box A minus 2 pounds is three times as heavy as box B
- The weight of box B minus 3 pounds is three times as heavy as the difference between C and D
- The weight of box C minus 4 pounds is three times as heavy as box D
- The weight of box D minus 5 pounds is 12 times less than the combined weight of box B and C

What are the weights of the 4 boxes?

Review of Foundational:

1) $[-10 \quad 2 \quad 7]+[10 \quad 12 \quad 2]=$ $[0 \quad 14 \quad 9]$

2) $\begin{bmatrix}5 & 2 & 5\\1 & 3 & 7\end{bmatrix}*\begin{bmatrix}1\\2\\3\end{bmatrix}=$ $\begin{bmatrix}24\\28\end{bmatrix}$

$2\times3 \quad 3\times1$

3) *What are the dimensions of the following matrix?*

$$\begin{bmatrix}5 & 3 & -12\\6 & 10 & 5\end{bmatrix}$$ $\boxed{2\times3}$

4) $\begin{bmatrix}144 & 5w\\12f & 128\end{bmatrix}=\begin{bmatrix}x^2 & 25\\72 & 3^r\end{bmatrix}$ $x=12 \quad w=5 \quad f=6 \quad r=4.42$

5) The determinant is $\frac{1}{ad-bc}$; find the determinant of the given matrix $\begin{bmatrix}4 & 3\\7 & 8\end{bmatrix}$ ✓

6) The Inverse of a 2 by 2 is given by $\frac{1}{ad-bc}*\begin{bmatrix}d & -b\\-c & a\end{bmatrix}$. Find the Inverse of $\begin{bmatrix}6 & 5\\-2 & 12\end{bmatrix}$ ✓

7) Subtract: $\begin{bmatrix}12 & 5 & 8\\3 & 2 & 1\end{bmatrix}-\begin{bmatrix}-2 & 7 & -4\\10 & 12 & 14\end{bmatrix}=$ ✓

8) $10*\begin{bmatrix}3 & 1\\4 & 2\end{bmatrix}+\begin{bmatrix}-2 & 1\\4 & 8\end{bmatrix}=$ ✓

For the following problems, perform matrix multiplication. If it is not possible, then write not possible.

9) $\begin{bmatrix}4 & 1 & 3\\-2 & 5 & 7\end{bmatrix}*\begin{bmatrix}1\\8\\10\end{bmatrix}=$ $[2\times3][3\times1] = \begin{bmatrix}42\\108\end{bmatrix}$

10) $\begin{bmatrix}4 & 12\\1 & 18\end{bmatrix}*\begin{bmatrix}2 & 0\\1 & 5\end{bmatrix}=$ $\begin{bmatrix}20 & 60\\20 & 90\end{bmatrix}$

11) $\begin{bmatrix}5 & 2\\1 & 6\end{bmatrix}*\begin{bmatrix}-2 & 1\\5 & 8\\10 & 12\end{bmatrix}=$ $(2\times2)(3\times2)$ Not possible

12) Solve the following system. This is not a high challenge problem. You may use the determinant as a way to help you find the inverse and then simply multiple the inverse by both sides.

$\begin{bmatrix}1 & 1\\2 & 18\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-6\\-140\end{bmatrix}$ $\frac{1}{16}\begin{bmatrix}18 & -1\\-2 & 1\end{bmatrix}$ $\begin{bmatrix}\frac{18}{16} & -\frac{1}{16}\\-\frac{2}{16} & \frac{1}{16}\end{bmatrix}\begin{bmatrix}-6\\-140\end{bmatrix}=\begin{bmatrix}2\\-8\end{bmatrix}$

We spent 2 days working on HC matrix problems. Those will be graded on showing all the work and all the necessary notation to properly communicate how you solve a matrix system. All work must be shown. You can look on my website for some additional support. For the video that asks you to sign in, just use your mpls credentials.

(5) $\frac{1}{11}$

(6) $\frac{1}{82}\begin{bmatrix}12 & -5\\2 & 6\end{bmatrix}$

$\begin{bmatrix}\frac{12}{82} & \frac{-5}{82}\\\frac{2}{82} & \frac{6}{82}\end{bmatrix}$

(7) $\begin{bmatrix}14 & -2 & 12\\-7 & -10 & -13\end{bmatrix}$

(8) $\begin{bmatrix}30 & 10\\40 & 20\end{bmatrix}+\begin{bmatrix}2 & 1\\4 & 8\end{bmatrix}=\begin{bmatrix}28 & 11\\44 & 28\end{bmatrix}$

** ** Examples of Problems that Require Significant Work no shortcuts. — Know

(1) Show All work to solve

$$\begin{bmatrix} 13 & -6 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ 23 \end{bmatrix}$$

Define: Let $[A] = \begin{bmatrix} 13 & -6 \\ 2 & 3 \end{bmatrix}$ And $[B] = \begin{bmatrix} 22 \\ 23 \end{bmatrix}$

then

$$\cancel{[A]^{-1}[A]} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = [A]^{-1}[B]$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = [A]^{-1}[B]$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{51} & \frac{6}{51} \\ \frac{-2}{51} & \frac{13}{51} \end{bmatrix} \cdot \begin{bmatrix} 22 \\ 23 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

How to find Inverse of 2 by 2

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 13 & -6 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$13a + 2b = 1 \; (3)$

$-6a + 3b = 0 \; (2)$

$39a + \cancel{6b} = 3$

$+12a \; \cancel{-6b} = 0$

$51a = 3$

$a = \frac{3}{51}$

$2b = \frac{12}{51}$

$b = \frac{6}{51}$

$13c + 2d = 0 \; (3)$

$-6c + 3d = 1 \; (2)$

$39c + 6d = 0$

$+12c \; -6d = 2$

$51c = -2$

$c = \frac{-2}{51}$

$2d = \frac{26}{51}$

$d = \frac{13}{51}$

(2)
$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 16 \end{bmatrix}$$

$$\begin{array}{ccc:c} 2 & 3 & -1 & 9 \\ 1 & 1 & 1 & 9 \\ 3 & 2 & 1 & 16 \end{array}$$

switch R_1 & R_2

$$\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 2 & 3 & -1 & 9 \\ 3 & 2 & 1 & 16 \end{array}$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 1 & -3 & -9 \\ 3 & 2 & 1 & 16 \end{array}\right] \Rightarrow \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 1 & -3 & -9 \\ 0 & -1 & -2 & -11 \end{array}\right] \Rightarrow \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 1 & -3 & -9 \\ 0 & 0 & -5 & -20 \end{array}\right]$$

so $\begin{bmatrix} x = 2 \\ y = 3 \\ z = 4 \end{bmatrix}$