Advanced Algebra

Binomial Expansions.

$\boxed{\text{Assignmt \#1}}$

Note: The sum of (a+b) is called a binomial as it contains two terms. Any expression of the form $(a+b)^n$ is called a power of a binomial.

$(a+b)1= a+b$

$(a+b)^2=a^2+2ab+b^2$

$(a+b)^3=(a+b)(a+b)^2$ which is $(a+b)(a^2+2ab+b^2)$ which is $a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$ which is $a^3+3a^2b+3ab^2+b^3$

$(a+b)^2 = a^2+2ab+b^2$

$(a+b)^3= a^3+3a^2b+3ab^2+b^3$

	a^2	$2ab$	b^2
a	a^3	$2a^2b$	ab^2
b	a^2b	$2ab^2$	b^3

To do:

1) Expand $(a+b)^4$ in the same way as $(a+b)^3$ above

$(a+b)(a+b)^3 \quad (a+b)(a^3+3a^2b+3ab^2+b^3)$

$a^4+3a^3b+3a^2b+ab^3+ba^3+3a^2b^2+3ab^3+b^4$

2) Then expand algebraically $(a+b)^5$ using your expansion for $(a+b)^4$ above $\boxed{a^4+4a^3b+6a^2b^2+4ab^3+b^4}$

$(a+b)(a^4+4a^3b+6a^2b^2+4ab^3+b^4)$

$\cancel{a^5}+\cancel{4a^4b}+\cancel{6a^3b^2}+\cancel{4a^2b^3}+\cancel{ab^4}+\cancel{ba^4}+\cancel{4a^3b^2}+\cancel{6a^2b^3}+\cancel{4ab^4}+b^5$

3) Expand $(a+b)^6$ using your expansion for $(a+b)^5$ $\boxed{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5}$

$(a+b)(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5)$

$\cancel{a^6}+\cancel{5a^5b}+\cancel{10a^4b^2}+\cancel{10a^3b^3}+\cancel{5a^2b^4}+ab^5$

$\cancel{ba^5}+\cancel{5a^4b^2}+\cancel{10a^3b^3}+\cancel{10a^2b^4}+5ab^5+b^6$

4) The $(a+b)^3= a^3+3a^2b+3ab^2+b^3$ expansion contains 4 terms:

a^3, $3a^2b$, $3ab^2$ and b^3. The coefficients are 1 3 3 1

a) What happens to the powers of a and b in each term of the expansion of $(a+b)^3$?

b) Does the pattern in a continue for the expansions of $(a+b)^4$, $(a+b)^5$ and so on?

$\boxed{a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6}$

c) Write down the triangle of coefficients to row 6:

n=0	1
n=1	1 1
n=2	1 2 1
n=3	1 3 3 1 ← row 3

5) The triangle of coefficients in c above is called Pascal's triangle.

n=4	1 4 6 4 1
n=5	1 5 10 10 5 1

6) Predict the elements of the 7th row of Pascals triangle.

n=6	1 6 15 20 15 6 1
n=7	1 7 21 35 35 21 7 1

Hence, write down the binomial expansion of (a+b)^7

$a^7+7a^6b+21a^5b^2+35a^4b^3+35a^3b^4+21a^2b^5+7ab^6+b^7$

Check your result algebraically by using $(a+b)^7 = (a+b)(a+b)^6$ and your results from **#3**