

Advanced Algebra

Unit 7 Binomial Expansion Assignment #5

1) Write down the first 3 terms (so find 0, term 1 and term 2) and the last 2 terms of the following:

a)  $(1+2x)^{11}$

$$1 + 11 \binom{10}{1} (2x) + 55 \binom{9}{2} (2x)^2$$

First 3 Terms  
 $1 + 22x + 220x^2$

$$\binom{11}{10} 1 (2x)^{10} + (2x)^{11}$$

Last 2 Terms:  
 $11264x^{10} + 2x^{11}$

b)  $(3x + \frac{2}{x})^{15}$

$$(3x)^{15} + \binom{15}{1} (3x)^{14} (\frac{2}{x}) + \binom{15}{2} (3x)^{13} (\frac{2}{x})^2$$

First 3 Terms:  
 $14348907x^{15} + 143489070x^{13} + 669615660x^{11}$

Last 2 Terms:

$$\binom{15}{14} 3x (\frac{2}{x})^{14} + (\frac{2}{x})^{15}$$

$$\frac{45x \cdot 2^{14}}{x^{14}}$$

Last 2 Terms:  
 $\frac{737280}{x^{13}} + \frac{2^{15}}{x^{15}}$

c)  $(2x - \frac{3}{x})^{20}$  Hint: Remember the signs alternate

$$(2x)^{20} - \binom{20}{1} (2x)^{19} (\frac{3}{x}) + \binom{20}{2} (2x)^{18} (\frac{3}{x})^2$$

Last 2 Terms:  
 $\binom{20}{19} 2x (\frac{3}{x})^{19} - (\frac{3}{x})^{20}$

2) Write down

a) the 6<sup>th</sup> term of  $(2x+5)^{15}$

$$\binom{15}{6} (2x)^9 5^6$$

$$\frac{15!}{9!6!} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10}{6 \times 5 \times 4 \times 3 \times 2}$$

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6<sup>th</sup> term

b) the 4<sup>th</sup> term of  $(x^2+y)^9$

$$\binom{9}{4} (x^2)^5 y^4$$

4<sup>th</sup> term

c) the 10<sup>th</sup> term of  $(x - \frac{2}{x})^{17}$

$$- \binom{17}{10} x^7 (\frac{2}{x})^{10}$$

+ - + - + - + - + -

10<sup>th</sup> term

d) the 9<sup>th</sup> term of  $(2x^2 - \frac{1}{x})^{21}$

$$\binom{21}{9} (2x^2)^{12} (\frac{1}{x})^9$$

9<sup>th</sup> term

3) Find the coefficient of  $x^5$  in the expansion of  $(x+2)(x^2+1)^8$

$$(x+2) \left[ (x^2)^8 + {}_8C_1 (x^2)^7 \cdot 1 + {}_8C_2 (x^2)^6 \cdot 1^2 + {}_8C_3 (x^2)^5 \cdot 1^3 + {}_8C_4 (x^2)^4 \cdot 1^4 + \dots \right]$$

$$\boxed{{}_8C_0 (x^2)^2 \cdot 1^6}$$

$$\frac{8!}{2!} \boxed{28x^5}$$

when  $\bar{2}$  mult, ply this by  $x \bar{2}$  get  $x^5$

4) Find the constant term in

a) the expansion of  $(x + \frac{2}{x^2})^{15}$

$${}_{15}C_r x^{15-r} \left(\frac{2}{x^2}\right)^r$$

$$x^{15-r} \cdot \frac{2^r}{x^{2r}}$$

$$x^{15-3r} \cdot 2^r$$

I want

$$15-3r = 0$$

$$\boxed{r=5}$$

so

$${}_{15}C_5 x^{10} \frac{2^5}{x^{10}}$$

$$\boxed{{}_{15}C_5 2^5}$$

b) the expansion of  $(x - \frac{3}{x^2})^9$

$${}_{9}C_r x^{9-r} \left(\frac{3}{x^2}\right)^r$$

$$x^{9-r} \cdot \frac{3^r}{x^{2r}}$$

$$x^{9-3r} \cdot 3^r$$

$\begin{matrix} + & - & + & - & + & - & + & - & + & - \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & - \end{matrix}$

so I need  $x$  to be to 0 so I AM looking

for  $x^{9-3r} = 0$   $\boxed{r=3}$

$$\boxed{+ {}_9C_3 \cdot 3^3}$$

$$\boxed{2268}$$